FORWARD TO THE PAST

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I am very grateful to the Tomalla Foundation for Gravity Research for the Tomalla Prize 2009 and consider awarding me with this prestigious prize as a great honour for me. Indeed, the height of the scientific level of this prize have been already determined by its first two emblematic recipients: Prof. Subrahmanyan Chandrasekhar and Acad. Andrei Sakharov.

Once these names are mentioned, I cannot avoid reminiscences. I was very fortunate to meet both these great scientists and great men, and not simply to meet but to have a possibility to discuss many things with them. And they knew my works, too, as you will see soon.

As for Prof. Chandrasekhar, I first met him in Moscow in 1983 or 1984 (I cannot remember the exact date now) when he came to Moscow for a few days going to Armenia, to the Byurakan observatory, to his old friend Acad. Ambartsumian. I used the following way to introduce myself to him. A few years before this, Prof. Chandrasekhar had become interested in the theory of small perturbations on black hole backgrounds, including the Kerr one. Evidently having been impressed by some results of my 1983 paper (in collaboration with Semyon Churilov, but this part was mine), he introduced the notion of ’Starobinsky constants’ in his journal papers and in his famous book ”The mathematical theory of black holes”, notably, without ever meeting me before that. So, naturally I used this favourite chance, came up to him and said, smiling: ”Prof. Chandrasekhar, I am the same Starobinsky whose constants you introduced.” You may guess what was the reaction. By the way, he noted that he had thought I should be older. His more deep question was if I could explain why these constants exist at all. What he meant is if I can explain the structure of these non-trivial constants without a direct calculation – my answer was ”No, I can’t” (it would remain the same now). Though this topic is not connected to inflation and cosmological perturbations, let me show you the expressions which interested him. These constants arise when solving the Teukolsky equations for electromagnetic and gravitational perturbations of rotating black holes and relating expressions for different quantities in the Newman-Penrose formalism.

’Starobinsky constants’ A_s:

$$\begin{gathered}A_0 = 1\ ,\\ 16A_1^{-1} = (\lambda - 2am\omega + 2)^2 + 4a\omega(m - a\omega)\ ,\\ 256A_2^{-1} = [(\lambda - 2am\omega + 6)^2 + 4a\omega(m - a\omega)][(\lambda - 2am\omega + 4)^2 + 36a\omega(m - a\omega)]\\ -48a\omega(m - a\omega)(2\lambda - 4am\omega + 11) + 144\omega^2(M^2 - a^2)\ ,\end{gathered}$$

where $\lambda = \lambda(s, a\omega)$ is an eigenvalue of the equation for spheroidal harmonics:

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{dP}{d\theta}\right) - \left(a^2\omega^2\sin^2\theta + \frac{n^2}{\sin^2\theta} + 2a\omega s\cos\theta + \frac{2ms\cos\theta}{\sin^2\theta} + \frac{s^2}{\tan^2\theta} - s\right)P + \lambda P = 0\ .$$

My first encounter with Andrei Dmitrovich Sakharov was much earlier – already in 1971. At that time, I was an undergraduate student of Acad. Yakov Borisovich Zeldovich, and we had just made a paper (a top-cited one by now) about particle creation and vacuum polarization in an external gravitational field including the calculation of the renormalized energy-momentum tensor in the strong anisotropic, but homogeneous case (the Bianchi I type metric). By the way, our method of renormalization was equivalent to what is called the adiabatic renormalization now (we called it 'the n-wave method'). Zeldovich proposed me to go to Sakharov and explained this work to him. So, I visited him in his Moscow flat. He listened favourably, with eyes semi-closed, no critical remarks, though my impression was that his face showed "I knew all this myself more or less". Indeed, the simplified form of our expressions for the energy-momentum tensor is that they are proportional to $\hbar c^{-3}t^{-4}$ near a curvature singularity where the metric behaviour is typically a power-law one (up to a logarithmic multiplier arising, due to renormalization, in the vacuum polarization part, but not in the part describing real created particles). Thus, the whole effect has approximately the form of corrections to the Einstein equations, which are second order in the Riemann tensor, which he envisaged in his famous 1967 paper on the induced gravity approach.

After that we met several times, both before and after his exile. In particular, I remember that meeting him first time after his return from the exile, I asked him, may be a little bit provocatively, "Andrei Dmitrovich, you might forget me since our previous meeting". He smiled, understanding my game immediately, and answered: "No, not at all. Andrei (Linde) always told me about your works on inflation when visiting me". (Andrei Linde, as well as some other Sakharov's colleagues in the Lebedev Institute were permitted to visit him in exile several times). In fact, he mentioned my name in his memoirs very favourably (though pointing that he would prefer a different variant of inflation, namely that with a constant vacuum energy and a constant Hubble parameter H – I shall return to this point later).

Now I am getting closer to the area of investigations for which the Tomalla Prize 2009 has been awarded to Prof. Mukhanov and me. Just since from 1971 Zeldovich and me began to think how it might be possible to find any observational consequences of the particle creation effect. At one moment early in 1972 it even seemed to us that it was possible to generate the flat spectrum of primordial scalar (density) perturbations using phonon creation in the radiation dominated early Universe, but it was - alas! - a result of an error in calculations made in a hurry, and we realized that such a spectrum was strongly blue tilted – so, completely uninteresting. Fortunately, this error was reflected only in one abstract of talks presented at the Russian Gravitational conference in 1972 (in Erevan). After that, Zeldovich very soon wrote his famous 1972 MNRAS paper where he proposed the flat spectrum (now called the Harrison - Zeldovich one) simply as a hypothesis. (Strictly speaking, this is an oversimplification – he *did* proposed some model for this, as well as Harrison did, too, but both these models are wrong actually, and their authors never insisted on them afterwards).

Regarding gravitons, we understood from our 1971 results that graviton particle creation may result in some amount of non-thermal and anisotropic graviton component with the energy density comparable to that of the cosmic microwave background but with the very high frequency $\sim 10^{11}$ Hz, which seems to be not observationally achievable even at the present time. Also, after Leonid Grishchuk showed in his 1974 paper that the amount of gravitons created in a purely isotropic, FRW, case is of the same order as in the anisotropic one, I proposed in 1976 to use the slope of the primordial gravitational wave spectrum to determine what was before the singularity (assuming that it was regularized by some unknown mechanism). However remarkable it might seem in principle, in practice it follows from the formula I obtained that, for natural models of contraction before the bounce, the effect is very small. So, I published this result in the abstract book of some conference only, until 1979, when I reproduced it in my 1979 JETP Lett. paper to show its crucial difference with the result for the de Sitter (later dubbed inflationary) beginning.

So, though the de Sitter beginning was known in principle (in USSR, at least) due to the Gliner's hypothesis made as early as 1965 (but it was considered as non-realizable and non-testable even by Sakharov and Zeldovich, see e.g. the footnote in the famous Sakharov 1965 JETP paper where the baryon acoustic oscillations were predicted which were discovered in 2005), I began interested in it, and in the gravitational particle creation once more, in 1979 when I obtained the result that in this case gravitational particle creation may lead to really observable effects at large, and even cosmological scales. Let me remind here a very simple and elegant main result of my 1979 JETP Lett. paper for the spectral density of gravitons created during the initial de Sitter (inflationary) stage with the Hubble parameter H_I after its decay to the radiation-dominated (RD) stage:

$$\omega \frac{d\epsilon_g(\omega)}{d\omega} = \frac{2GH_I^2}{3\pi}\epsilon_{rad}$$

– thus, the flat energy spectrum. Here radiation is assumed to be self-interacting, i.e. $p_{rad} = \epsilon_{rad}/3$, but the correction due to possible admixture of a free particle component is numerically small. Also, the Einstein gravity is assumed, and the coefficient is parametrically different for scalar-tensor and $f(R)$ gravity, as I found afterwards.

This spectrum corresponds to a very non-thermal occupation number of gravitons with $\omega \ll 10^{11}$ Hz: $n(\omega) \propto \omega^{-4}$. Note the subtle point regarding this formula which is sometimes missed: it contains the additional 1/2 multiplier due to the fact the this GW background consists of standing waves only (in quantum terminology, its density matrix is strongly squeezed in phase space). Another subtle point (which continues to confuse some people even now, including such a pioneer of particle creation in cosmology as Prof. Leonard Parker) is that, of course, the renormalization is needed to obtain this result, but it may be achieved by a very simple prescription – simply by omitting the so-called decaying mode of GW perturbations and keeping only the constant (quasi-isotropic) one after the first Hubble radius crossing during the de Sitter stage (I agree, however, that the rigorous proof and detailed explanation of this result requires many additional papers).

Another intermediate result (which was even not explicitly written in my paper for brevity but used in it, of course!) is the total average value of gauge-invariant tensor metric perturbations during the de Sitter expansion itself:

$$< h_{\alpha\beta}h^{\alpha\beta} >= \frac{16GH_I^3(t-t_0)}{\pi}$$

where $\alpha, \beta = 1, 2, 3$ and t_0 is the moment of the (local) beginning of the de Sitter expansion. It is interesting that consequences of this expression remain not fully understood up to the present time. Indeed, it shows that the exact de Sitter space-time may be not be considered stable with respect to GW since the quantity in the right-hand side becomes of the order of unity for $t - t_0 \sim G^{-1}H_I^{-3}$, so that the assumption of small deviations from the global de Sitter space-time breaks down. There exist different opinions about what happens in this (may be, a little bit academic) case of the Einstein gravity plus a stable cosmological constant. My opinion (still has to proved rigorously) based on my 1983 result about a late-time attractor for the classical version of this case is that an eternal stochastic drift occurs through an infinitely degenerate set of 'vacuum' states which are locally de Sitter-like but globally not de Sitter at all.

Using these results for metric perturbations, it is straightforward (though it requires a rather large amount of work) to derive predictions for CMB temperature anisotropy and polarization generated by these primordial GW. First, it was numerically obtained by Rubakov, Veryaskin and Sazhin in 1982 for $l = 2$ and $l = 3$ multipoles. Let me remind you my analytical results for the flat 'Sachs-Wolfe' plateau (C_l with $2 \ll l < 50$) and for the total anisotropy published in my 1985 Astron. Lett. paper, since getting them requires some art in dealing with integrals of transcendental functions which cannot be found in standard tables of integrals. Actually, I have never published the full derivation, and nobody did it afterwards, too, but numerical calculations confirmed them!

$$\langle \left(\frac{\Delta T}{T}\right)^2_{lm} \rangle = B^2 \left(l + \frac{1}{2}\right)^{-2} \frac{1}{36\pi} \left(1 + \frac{48\pi^2}{385}\right) ~,$$

$$\langle \left(\frac{\Delta T}{T}\right)^2 \rangle = \sum_{l=2}^{\infty} \frac{2l+1}{4\pi} \langle \left(\frac{\Delta T}{T}\right)^2_{lm} \rangle = \frac{B^2}{72\pi^2} \left[\left(1 + \frac{48\pi^2}{385}\right) \ln \frac{\eta_0}{\eta_{rec}} - \frac{432}{385}\,\zeta(3) + \frac{232}{1155} \right] ~,$$

where $B^2 = 16\pi G H_I^2$, and η_0 and η_{rec} are the present moment and the recombination moment in terms of the conformal time (the present cosmological constant is assumed to be zero here but its presence changes the results for a few percent only).

So, one need not construct any inflationary model to calculate the GW primordial spectrum. However, this is not so for the most interesting class of perturbations: the scalar (density) ones. To get a meaningful (i.e. finite) answer, one has to consider some concrete model of decay of a primordial de Sitter (inflationary) stage to the subsequent RD stage (possibly through an intermediate matter-dominated (MD) stage). Note that this point is not trivial at all, and it continues to confuse some well known people even today. I realized this and began to think about some internally self-consistent model of this type. Ironically, I had almost had this model at hand since my 1978 Astron. Lett. paper where an analytical description of a non-singular bouncing closed FRW model driven by a scalar field with the $m^2\phi^2/2$ potential was given. In fact, this model contains two inflationary stages – contracting and expanding ones, and what is called the slow-roll approximation now was first introduced in this paper. Actually, the decisive step which Andrei Linde did in 1983 was simply to throw away the whole contracting stage (and then the assumption of a closed FRW model may be lifted, too).

However, I met with serious problems with this paper: it was rejected by JETP (the only such case in my scientific career), and the main argument was just that one which was repeated many times against chaotic inflation and remains by now: why so large and homogeneous scalar field exceeding $M_{Pl} = G^{-1}$? At present our answer still remains mostly "Why not?", some people do not worry about it, some do. Even Zeldovich was not happy with this paper (finally it was published in a shorter version in Astron. Lett.). However, this bouncing model is still generic in the sense that it has a non-zero measure in the space of all initial conditions for this toy Lagrangian. As a result, I shifted from a (quasi-)classical scalar field to the fourth-order gravity with one-loop quantum-gravitational corrections and found (the paper was submitted to Phys. Lett. in January 1980) the desired internally consistent model with slow-roll during the quasi-de Sitter expansion, with a graceful exit to the subsequent MD stage driven by a scalar (quasi-)particle appearing in this theory of gravity and with sufficiently effective 'reheating', namely, with a mechanism of decay of these scalarons into pairs of particles and antiparticles of all kinds of quantum matter fields. Thus, this model had already contained all necessary building elements of inflationary models. Under some assumption about the relation between parameters of this model, its main part is simply the particular (and the simplest non-trivial) case of the so called $f(R)$ gravity with $f(R) = R + R^2/6M^2$ where M is just the scalaron mass in flat space-time (M should be taken $\approx 3 \times 10^{-6} M_{Pl}$ to fit observational data). It is interesting that gravitational particle creation is extremely important for this model: not only does it generate required perturbations, but all observed matter in the Universe appears through this mechanism.

In my 1980 paper, only isotropic (FRW) solutions of all 3 kinds were studied. To derive spectra of scalar and gravitational perturbations in this model, and to prove its stability against them, first, I supposed, equations for small perturbations in this model should be derived (which are non-local, in fact, if one-loop corrections are taken into account). This required large work, so I published the resulting equations in second half of 1981 only. However, it appeared that one could obtain the correct value of the scalar slope n_S in this model without all these complications (but assuming stability), and here Slava Mukhanov and Gena Chibisov were quicker and did it in the beginning of 1981. Still these equations were necessary to prove the very possibility of a graceful exit from inflation, and to obtain correct values of the amplitudes of scalar and tensor perturbations. The final results for this model first appeared in my 1983 paper in Astron. Lett., since in 1982 most of proponents of inflation were busy with the first viable scalar field inflationary model – the 'new' inflation – and with the calculation of the spectrum of scalar perturbations in it – the dramatical history of the Cambridge Nuffield symposium in the summer of 1982 when the 3 papers: by Steven Hawking, myself and Alan Guth with So-Young Pi were submitted in this order of succession with less than one month difference in between. In all 3 papers the correct slope of the scalar spectrum for the 'new' inflation was obtained, and in the two latter ones - the amplitude, too (as for the GW power spectrum in this model, one may use my 1979 result).

Let me now skip the following history (which had many dramatic moments, too) and show you my very old transparency (Fig1.pdf) with the final prediction of inflation regarding perturbations written about 20 years ago – that is why I apologize that it is slightly dirty, but for me, it is historic. I used it so long since there was no necessity to change anything (I added the modern definitions of ζ, P_ζ, P_g and r only). What written in the lower part of this transparency is the expression for perturbations in terms of the difference of the duration of an inflationary stage in different points of space which constitutes the basis of the so called δN-formalism which is so successively used now in many papers. Moreover, this expression remains valid if perturbations are not small and the spatial part of the space-time metric is written in the form $a^2(t)\exp(h(\vec{r}))dx_\alpha dx^\alpha$ (in the case of a single, one-field inflation, such relation appeared already in the same my 1982 paper on perturbations in the 'new' inflation). Let me shortly mention also that in the case of scalar perturbations, in contrast to GW, we know how to treat their generation and their back-reaction on a background space-time metric in the regime when $|h|$ is not small. This is achieved using the stochastic approach to inflation (shortly called 'stochastic inflation') fully developed in my 1984 paper (existing in Russian only – my first paper in English with this method appeared in 1986). Stochastic inflation has many remarkable consequences. In particular, it leads to the notion of 'eternal inflation' in the Meta-Universe (or, 'multiverse') introduced by Andrei Linde in 1986. However, since inflation in the observable part of our Universe is not eternal, it had both its beginning and its end in the past, it is not clear at present which directly observable predictions of eternal inflation exist, if any. So, I leave this fundamental and breathtaking topic and would like to say a few words about observational discoveries expected in future, especially after the recent successful launch of the Planck satellite.

First of all, one discovery is *guaranteed* at any circumstances: the determination of the slope n_s of the primordial power spectrum of scalar (density) perturbations as a function of k. At present, we know only that $n_s(k) - 1$ is small and its value, averaged over the observed interval of scales, is slightly less than unity ($\sim 0.96 - 0.97$). Measurement of this function will greatly reduce the number of still possible inflationary models. However, as I explained above, another fundamental and observable prediction of inflation is the primordial GW background, which may be sufficiently large to be observable for some inflationary models (though not for all of them). Let us look at my another transparency (Fig2.pdf) with the discussion of how much primordial GW could we expect. These expectations are classified using the (mainly observational) parameter N – the duration of the last, observable part of inflation:

$$N = \ln \frac{R_{ls}}{\lambda_\gamma} - \frac{1}{2} \ln \frac{M_{Pl}}{H_I} - \Delta N_{reh} \ .$$

Depending on the duration of reheating, N is typically in the range $50 - 60$. So, in the more optimistic case when $r/8 \sim N^{-1}$, primordial GW background will be certainly discovered by the Planck team. In this case, supposing the present average value of n_s be confirmed, the best inflationary model is the simplest scalar field one – that with $V = m^2\phi^2/2$ and $m \approx 1.4 \times 10^{-6} M_{Pl}$ (for $N = 50$). If the N^{-1} class of models will be falsified by the Planck and other ongoing experiments, the next class of models with $r/8 \sim N^{-2}$ (called 'less optimistic' in Fig2.pdf) becomes the most promising in this respect. Among them, the simplest ones (i.e. with only one dimensionless parameter) which predict the presently measured average value of n_s correctly are my 1980 $R + R^2/6M^2$ model and the Higgs inflation model recently proposed by Fedor Bezrukov and Mikhail Shaposhnikov (in the main approximation, these models are equivalent with respect to the description of inflation and predictions for primordial perturbation spectra). It is interesting to note that inflation in these models occurs with a practically constant Hubble parameter H in the Einstein frame in which the effective gravitational constant is constant by definition (while H slowly changes with time in the physical, Jordan frame). Thus, returning to the remark in the Sakharov's memoirs mentioned above, a difference between our favourite models for inflation may finally appear to be frame-dependent, i.e. inessential.

Now let me emphasize three main things about inflation, perturbations and gravitational particle creation.

1. Inflation and gravitational particle creation help and promote each other greatly. Without inflation, investigation of gravitational particle creation would mostly remain an academic exercise (apart from a possibility of the Hawking radiation from small primordial black holes which have not been found). Without gravitational particle creation, inflation will mostly remain without any new and positive observational predictions, and could only 'explain' already known things.

2. In the scope of the inflationary scenario, observed inhomogeneities represent a genuine quantum-gravitational effect (although greatly amplified by quasi-classical instabilities like those due to which we can 'see' individual elementary particles in accelerators). This is completely evident in the case of graviton production during inflation, but it refers to the generation of observed scalar perturbations, too, since to calculate their spectrum in an internally consistent way, one *should* quantize a space-time metric.

3. Since perturbations (and the whole present Universe, in fact) are quantum, but observed astronomical objects are classical, a theory of the quantum-to-classical transition for perturbations is required. As was emphasized in my 1996 paper with David Polarski, the same omission of the decaying mode which is necessary for renormalization effectively brings us the desired transition without necessity to consider any concrete model of de-

coherence (or any other alternative). Perturbations, as well as the Universe as a whole, formally remain quantum even at present, but we may describe them in terms of *stochastic* classical variables with an excellent accuracy (less than 10^{-100} relative error for cosmological scales). However, would one wish to go deeper and understand why such and not any other realization of these effective stochastic quantities has occurred, did God play dice with them or not, one has to think about foundations of quantum mechanics, and inflation and particle creation bring nothing principally new to this fundamental problem. In other words, I suppose that primordial cosmological perturbations represent a similar, but much more interesting and much more actual object for those who want to study these things than the famous "Schrödinger cat".

Finally, I would like to illustrate dramatic changes which occurred in cosmology during the last 30 years by presenting how notions about the whole history of our Universe, compressed into one line, were changing.

1. Before 1980.

$$FRWRD \Longrightarrow FRWMD$$

2. After 1980.

$$DS \Longrightarrow FRWRD \Longrightarrow FRWMD$$

3. After 1998.

$$DS \Longrightarrow FRWRD \Longrightarrow FRWMD \Longrightarrow \overline{DS}$$

But I suppose that the correct history is such one:

$$? \longrightarrow DS \Longrightarrow FRWRD \Longrightarrow FRWMD \Longrightarrow \overline{DS} \longrightarrow ?$$

Indeed, let us exploit the remarkable *qualitative* analogy between DS and $\overline{DS}$. Let us read the line in the third paragraph from right to left. There surely was a long history before $\overline{DS}$, why should we expect something different regarding DS? Now let us read it in the opposite direction: DS was surely unstable, why $\overline{DS}$ should be different?

Anyway, the development of the inflationary scenario and the theory of generation of perturbations during it has provide us with a unique possibility to make one more step forward to the past of our Universe (this explains the title of my lecture). Now I suppose it is the proper time to make the next step and try to discover what was before DS, not by insisting that there may be only one 'wave function of the Universe' (in what I don't believe), but to look for principally new observable effects depending, first, on the whole duration of the local inflationary stage (DS) and, second, on a pre-inflationary state (during which what that becomes our observable Universe after inflation was only a part of a much larger and richer Meta-Universe, or 'multiverse'). It is clear that this task is extremely difficult, but I don't think it is completely impossible.

Thank you for your attention and thank the Tomalla Foundation once more.